Logical Arithmetic 2: The Sorites

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Rough Draft

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1. Introduction

A “sorites” is a chain argument. It involves the connection of several propositions into one long argument. A sorites that begins with a major premiss (the top one) is called a Goclenian sorites. If it begins with a minor premiss (the bottom one), it’s called an Aristotelian sorites.

Example 1: Goclenian

Here is an example of a Goclenian sorites:

All Z is P...(note: major premiss)
All Y is Z
All X is Y
All S is X
∴ All S is P

In syllogistic notation, with boxes around the cancelled terms:

\[ \text{Ze} \overset{\sim}{\rightarrow} \text{P} + \text{Ye} \overset{\sim}{\rightarrow} \text{Z} + \text{Xe} \overset{\sim}{\rightarrow} \text{Y} + \text{Se} \overset{\sim}{\rightarrow} \text{X} = \text{Se} \sim \text{P} \] (or SaP).

Since some of the middle terms are suppressed or hidden, we will not say that we can cancel the middle terms of the above argument, but rather that we can cancel the “linking” terms. The terms that we can cancel are obviously the ones that are polarized, having a positive term operator and a negative term operator. These are Z, Y, and X. If a term is polarized, then any other occurrence of that term, whether in negative or positive form, can be cancelled as well.

Hence, if S is complemented somewhere down the argumentative chain by \( \sim S \), then further occurrences of S or \( \sim S \), however many there might be, can be cancelled. In the above equation, the remaining terms are \( \sim P \) and S. Combining these we have Se \( \sim P \). Since the remaining small letters in the equation are “ee,” the conclusion will have an “e” as well, then obverted to SaP.
Example 2: Aristotelian

Here is an example of an Aristotelian sorites:

All A is B…(note: minor premiss)
All B is C
All C is D
All D is E
\[ \therefore \text{All A is E} \]

In order to solve the above sorites, we want to get our cancelled terms next to the plus sign, rather than away from them. Here is the solution to the above Aristotelian sorites:

In syllogistic notation:

\[ Ae\sim B + Be\sim C + Ce\sim D + De\sim E = Ae\sim E \]
(or \(Aa\sim E\))

Compare this “Aristotelian equation” with the previous “Goclenian equation” and take note of the placement of the cancelled terms with reference to the + sign. The cancelled terms in the Aristotelian sorites are next to the plus sign, while the cancelled terms of the Goclenian sorites are away from the plus sign. To help the memory, here is a silly mnemonic: “Go clean away.” This mnemonic expresses that the Goclenian linking terms are away from the plus sign in deriving the conclusion.

2. More Examples

Example 3: Dogs & Snakes

Here’s an example of a Goclenian sorites from Parry and Hacker:

(1) All dogs are mammals.
(2) No reptiles are mammals.
(3) All snakes are reptiles.
(4) All cobras are snakes.
(5) Therefore, no dogs are cobras.

We would write this in syllogistic notation as:

\[ \sim MeD + ReM + Se\sim R + Ce\sim S = DeC \]

If you look very carefully at our equation, you will see at once how easy it is to solve soriteses. Simply set up the Goclenian sorites in the same way you’d set up a normal syllogism, then strike out all the polarized terms. Or, if you have an Aristotelian sorites, set it up in the same way you’d set up a transposed syllogism. And that’s all there is to it!

In the above equation, we can cancel \(M\), \(S\), and \(R\), and that leaves \(D\) and \(C\) as the uncancelled terms. Here’s how it would look with boxes around the linking terms:

\[ \sim MeD + ReM + Se\sim R + Ce\sim S = DeC \]

In this case, \(C\) is in the subject place, and \(D\) is in the predicate place, so our conclusion is \(CeD\), which is easily converted into \(DeC\), “No dogs are cobras.”

A Useful Tip: A Scorecard

If you’ve written a sorites down on paper and don’t want to box or pencil out the cancelled terms, or give your equation a sloppy look by marking through it, you can construct a scorecard by writing down the cancelled terms on one line of your paper, then the remaining terms on the next line; for example:
Cancelled terms:  M, S, R
Remaining terms:  D, C

This will help a great deal when we consider longer soriteses that involve dozens of cancelled terms.

**Example 4: Babies & Crocs**

Here is an Aristotelian sorites from Carroll:

(1) Babies are illogical.
(2) Nobody is despised who can manage a crocodile.
(3) Illogical persons are despised.
\[ \therefore \text{________________}. \]

Dictionary:  
- B = Babies
- l = logical
- c = manage a crocodile
- d = person despised

We shall symbolize the sentence operators, a,e,i,o using subscript form so they won’t be confused with lower case subject or predicate terms.

\[ B_a \sim l + d_c c + \sim l_d d = \]

or putting it in order:

\[ B_a \sim l + \sim l_d d + d_c c = \]

and solving:

\[ B_c l + \sim l_c \sim d + d_c c = \]

Cancelled terms:  l, d
Remaining terms:  B, c
Conclusion:  \( c_B (or \ B_c c); \) “No baby can manage a crocodile.”

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**Example 5: Wedding Cake**

Here’s a Goclenian sorites (again from Carroll):

(1) The only articles of food which my doctor allows me are such that are not very rich.
(2) Nothing that agrees with me is unsuitable for supper.
(3) Wedding cake is always very rich.
(4) My doctor allows me all articles of food that are suitable for supper.
\[ \therefore \text{______________}. \]

Dictionary:  
- P = permitted food
- R = rich
- A = agrees with me
- S = suitable for supper
- W = wedding cake

In syllogistic notation:

\[ P_e R + W_a R + A a S + S a P \]

And solving,

\[ R e P + W e R + ~ S e A + ~ P e S = \]

Cancelled terms:  P,R,S
Remaining terms:  A,W
Conclusion:  \( W e A, \) “No wedding cake agrees with me.”

---

**Example 6: Amos and Mutton**

Let’s try a longer one from Carroll:

(1) All the policemen on this beat sup with our cook;
(2) No man with long hair can fail to be a poet;
(3) Amos Judd has never been in prison;
(4) My doctor allows me all articles of food that are suitable for supper.
\[ \therefore \text{______________}. \]
(4) Our cook’s “cousins” all love cold mutton;  
(5) None but policemen on this beat are poets;  
(6) None but her “cousins” ever sup with our cook;  
(7) Men with short hair have all been in prison. 

Carroll’s dictionary:  

- a = Amos Judd  
- b = cousins of our cook  
- c = having been in prison  
- d = long-haired  
- e = loving cold mutton  
- h = poets  
- k = policemen on this beat  
- l = supping with our cook. 

We shall skip Carroll’s symbolization and render the above sentences into syllogistic notation (negative form):

Table 1: Using Syllogistic Notation fo a Sorites

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$k_e \sim l +$</td>
<td>$d_e \sim h +$</td>
<td>$a_e c +$</td>
<td>$b_e \sim e +$</td>
</tr>
<tr>
<td>5</td>
<td>$h_e \sim k +$</td>
<td>$\sim b_e l +$</td>
<td>$\sim c_e \sim d =$</td>
<td></td>
</tr>
</tbody>
</table>

Cancelled terms: k,l,d,h,b,c  
Remaining terms: a,~e 

The letter a can be found in the 3rd premiss as the subject term while the letter ~e can be found in the 4th premiss as the predicate term; hence the conclusion will be $a_e \sim e$, or $a_e e$; “All a is e,” which is Carroll’s conclusion, “Amos Judd loves cold mutton.”

In his book Symbolic Logic (Bartley edition), Carroll recommends the method of underscoring rather than canceling, but it amounts to the same thing. Hence, in solving the above sorites, Carroll takes the following equation:

$$k l \uparrow \uparrow d h \uparrow \uparrow a_1 c_0 \uparrow b_1 e_0 \uparrow k \sim h_0 \uparrow b \sim l_0 \uparrow \sim d \sim c_0 \uparrow \uparrow a_0$$

He then solves it by placing similar letters together and underlining. I’ve used **bold** to signify those letters that Carroll would have underlined twice (since I cannot symbolize two underlines). In Carroll’s method, one underline refers to a positive term, and the double underline refers to a complementary negative term, *i.e.*, k and $k'$, or in common notation, k and $\sim k$. (See Carroll’s, Symbolic Logic, p. 138ff., for a more in-depth discussion of his method.) Hence, the result would be:

$$k l \uparrow \uparrow k' h \uparrow \uparrow d h' \uparrow \uparrow b' l \uparrow b' e' \uparrow d' c' \uparrow a_0$$

The problem with this procedure is that it appears to double one’s efforts, when all that is required by our simplified logical arithmetic is to translate the argument into syllogistic notation. In fact, to speed things up, the argument can be converted into the negative form of common notation straight from the English sentences.

I believe logical arithmetic is a lot easier to use in solving such soriteses, much more than Carroll’s method, but it doesn’t hurt to know how to solve soriteses using different symbolisms. Still, the easier method is to be preferred, and I’ve also provided a simple way to
check for the validity of the conclusion (more later).

3. Multilateral Sorites

Carroll calls those sorites-premises that have complex subject terms or complex predicate terms “multilateral” premises, but for now, I’ll skip over explaining his method. (You can find Carroll’s exposition of it in his Symbolic Logic, starting with page 285.) Can our simplified logical arithmetic handle multilateral premises? I think it can. Here is a sentence from Carroll’s “Pork-Chop” problem that illustrates a multilateral premiss (that is, premises containing complex subjects or predicates):

“An earnest gambler, who is depressed though he has not been losing money, is in no danger of losing any.”

(In Carroll’s notation this is \( lma_1k_0 \).)

From the standpoint of logical arithmetic, the only difficulty these multilateral or complex premises give us is, first, the sheer number of terms that must be cancelled. But this is not a large problem, for it’s only a matter of taking the time to do it, writing down a scorecard to keep track, and so on. The primary difficult is in setting up the conclusion. Which remaining term goes where, in subject or in predicate place?

Example 7: Letters and tables

Now let’s have a look at one of Carroll’s soriteses with multilateral premises:

We can translate this into syllogistic notation:

We can translate this into syllogistic notation:

Cancelled terms: d,n,m,k,a,h,b,
Remaining terms: –c,e,l

Now the main problem is to find the proper form of the conclusion. In order to make this as simple as possible, let’s construct a schedule to find out whether
a term is subject or predicate. Here’s a simple rule: In a sorites involving multilateral premises, if two or more remaining terms occur with each other, their place in their own premiss determines their place in the conclusion. Seems a fairly simple rule. Let’s see how it works. In the following table the remaining letters e and l occur together in the 3rd premiss in subject position. They will therefore become the subject of the conclusion:

Table 4: A Useful Scorecard

<table>
<thead>
<tr>
<th>Premiss</th>
<th>S</th>
<th>Pr</th>
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</thead>
<tbody>
<tr>
<td>2</td>
<td>–c</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>e, l</td>
<td></td>
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<tr>
<td>8</td>
<td>e</td>
<td></td>
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<tr>
<td>5</td>
<td>l</td>
<td></td>
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</tbody>
</table>

The conclusion would be: el~c or by simple conversion ~c,el which is Carroll’s conclusion. In positive form, elc, “All e and l are c.”

Pretty darned simple! With regard to checking for validity (which we will discuss in more detail later), all that you need to do right now is make sure that for each premiss in the sorites, at least one term was cancelled.

Example 8: A Large Sorites

Let’s try a more complex sorites given by Carroll:

Table 5: Carroll’s Multilateral Argument

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<tr>
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<th>1</th>
<th>2</th>
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<tbody>
<tr>
<td>1</td>
<td>Cl₁E’₀ †</td>
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<td>2</td>
<td>Av₁D₀ †</td>
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<td>3</td>
<td>k₁m’₀ †</td>
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<td>4</td>
<td>lC’₁(b’n)’₀ †</td>
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We can simplify this by putting it in the
notation of logical arithmetic:

<table>
<thead>
<tr>
<th>Table 6: Multilaterals in Syllogistic Notation:</th>
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</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>$Cl_e \sim E$ +</td>
</tr>
<tr>
<td>5</td>
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<tr>
<td>$dsb_e t$ +</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>$e \sim m_e (rb)$ +</td>
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<tr>
<td>13</td>
</tr>
<tr>
<td>$D \sim n \sim r \sim b_e z$ +</td>
</tr>
<tr>
<td>17</td>
</tr>
<tr>
<td>$rD \sim h_e e$ +</td>
</tr>
<tr>
<td>21</td>
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<tr>
<td>$zt \sim B_e d$ +</td>
</tr>
</tbody>
</table>

This is where a scorecard really helps you keep track of things. If we go through the sorites and look at each letter we will find the following cancelled terms and remaining terms:

**Cancelled terms:** $CIEAvkmbstwraeHchByu$

**Remaining terms:** $dzD$

The scorecard would be:

<table>
<thead>
<tr>
<th>Table 7: Long Scorecard</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Premiss</strong></td>
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<td>6</td>
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</tbody>
</table>
The letters d and z occur together in the 24th premiss in the subject position, and will therefore be the subject term of the conclusion. The remaining letter D will be the predicate term.

Hence: \((dz)_eD\); No \((dz)\) is D. You could probably have skimmed over the argument very quickly and have seen almost immediately that the letters d and z occurred together in the 24th premiss, and if that is the case, you are under no obligation to construct a schedule as I have done above. You can build your conclusion as soon as you’ve got your remaining letters located. If you’re a genius, just skip the scorecard, but if you’re a bit of a dummy like me, it’s better to be safe than sorry. When the argument is long, it’s hard to see through all the clutter of letters for the requisite relationships, so I tend to write down a scorecard and just to be thorough. (A check of the sorites also shows that each premiss had a term that could be cancelled, so the conclusion was validly derived.)

4. Invalidity

The rules for invalidity are the same in Logical Arithmetic 2 as they are in Logical Arithmetic 1, especially the decision procedure for determining invalidity.

Example 9: Hobbits

Here’s an Aristotelian sorites from Parry & Hacker, with a negative premiss in the first place:

(1) No Orcs are Brandybucks.
(2) All Brandybucks are Shirefolk.
(3) All Shirefolk are Halflings.
(4) All Halflings are Hobbits.

If you need to construct a dictionary, it should look something like this, though you can certainly use any letters you want if it makes it easier: O = Orcs; B = Brandybucks; S = Shirefolk; L = Halflings; H = Hobbits. In syllogistic notation we have:

\(OeB + BaS + SaL + LaH =\)

Solving, \(OeB + Be~S + Se~L + Le~H =\)

Here it’s fairly obvious that we were unable to cancel the B’s, and this means there is a lack of distribution in the sorites.

Example 10: Hobbits Again

Let’s try the same one, with a negative premiss in the last place (before the conclusion):

(1) All Brandybucks are Shirefolk.
(2) All Shirefolk are Halflings.
(3) All Halflings are Hobbits.
(4) No Hobbits are Orcs.

\(BaS + SaL + LaH + HeO =\)

\(Be~S + Se~L + Le~H + HeO =\)

Cancelled terms: S, L, H, Remainning terms: B, O
Conclusion: BeO, “No Brandybucks are Orcs.”

The sorites is valid since we were able to cancel a term in each premiss. The possibility of unpolarized terms will
arise for soriteses with negative premises, but as long as your solution to a sorites involves at least one cancelled term in each premiss, you don’t need to worry about the placement of negative premises.

**Particular premises can lead to lack of distribution.** If the particular premiss does not connect up with a sentence where the linking term is connected to “All” or “No” then there will be a lack of distribution. Recall that an argument such as “Some S is P” and “Some P is Q,” therefore “Some S is Q” is not a valid conclusion. The “some” operator does not sufficiently distribute identity between sentences, so by itself cannot bring about validity in an argument. The presence of “some” in any type of sorites may cause the whole sorites to be invalid.

Moreover, if you have to use conversion by limitation to cancel the requisite number of terms, you will also have a particular premiss as a result. This may end up creating a lack of distribution in the sorites. The easiest way to handle this is to make use of rule 4 of our decision procedure. First, pick out the first two premises that were converted per accidens, then solve the equation and derive a conclusion. Second, combine this new conclusion with the very next premiss.

If you do this, and find that the relevant terms are not connected to “All” or “No,” then the sorites is invalid due to a lack of distribution (per rule 4). If, on the other hand, at least one of the cancellable terms is connected to “All” or “No,” the use of conversion by limitation has not resulted in a loss of distribution.

Having assured yourself that this use of conversion by limitation did not hurt the sorites, you should then go on to any other occurrence of conversion by limitation or of a particular statement. The very first one that evidences a lack of distribution will be all that is needed to show the invalidity of the sorites.

**Example 11: Aristotelian sorites**

Consider the following Aristotelian sorites:

\[
\begin{align*}
XaS \\
YaX \\
ZaY \\
PaZ \\
\therefore SiP
\end{align*}
\]

Solving, we obtain:

\[
So\sim X + Xo\sim Y + Yo\sim Z + Zo\sim P = So\sim P
\]

(or SiP).

We were able to cancel all the terms (next to the plus sign), but since we used conversion by limitation, we have one more step to go. Using rule 4 of our the decision procedure, we will seek to determine whether the relevant terms are connected to “All” or “No.” First, solve the first two premises that used conversion by limitation, then combine the resulting conclusion with the very next premiss:

The first two premises were converted to negative form and brought to the first figure by way of conversion by limitation, and this resulted in a particular conclusion:
This particular conclusion should then be combined with the very next premiss, ZaY:

\[
\begin{align*}
S_iY + ZaY &= \\
YiS + ZaY &= \\
Yo~S + Ze~Y &= ?
\end{align*}
\]

We’ve polarized the linking terms by using conversion, then obversion, but notice how the Y’s in the original argument are not connected to “All” or “No.” This demonstrates that the sorites is invalid. If the above equation had worked, however, you would need to go on to the next premises (if any) that used conversion by limitation, or a particular sentence, and test them in the same way. The first one that fails of distribution will show that the sorites is invalid. If there are no failures of distribution, the sorites is valid.

**Example 12: Particularity**

Here is the procedure for the occurrence of a particular statement:

\[
\begin{align*}
XaS \\
YiX \\
ZaY \\
PaZ \\
\therefore SiP
\end{align*}
\]

Solving, we obtain:

\[
So~X + Xo~Y + Yo~Z + Zo~P = So~P
\] (or SiP)

But are we entitled to this conclusion? Let’s check for lack of distribution. This is an Aristotelian sorites, and the second premiss is a particular statement, so we combine it with the first premiss and read the conclusion going left to right:

\[
So~X + Xo~Y = SiY
\]

This conclusion is valid, so we go on and combine it with the very next premiss:

\[
SiY + ZaY = ?
\]

As in the previous example, at least one linking term isn’t connected to “All” or “No.” This means the sorites is invalid.

**Example 13: Obversion**

Consider now the following sorites:

\[
\begin{align*}
XiS \\
YeX \\
ZaY \\
PaZ \\
\therefore SoP
\end{align*}
\]

Solving, we obtain:

\[
So~X + XeY + ~YeZ + ~ZeP = SoP
\]

Let’s check for distribution:

\[
So~X + XeY = SoY \\
SoY + ~YeZ = ?
\]

Now it would seem at first sight that we have a lack of distribution with SoY and ZaY. Remember, however, that rule 4 of our decision procedure says that if you can use obversion, followed by simple conversion (per rule 2), then this rule about distribution does not apply. This has to do, as we said, with the fact that you can negate the middle or the linking term prior to conversion, and this has the
effect of distributing the term. Hence ZaY becomes Ze~Y, then ~YeZ, and it can validly link with SoY. The sorites is valid.

The best thing to hope for is that you don’t come across any soriteses with particular propositions or propositions required to be solved through conversion by limitation, but if in fact you are unlucky enough to get one, the above procedure should help you to determine its validity.

A check of the logic table in our previous essay, “Logical Arithmetic,” shows that many syllogisms fail because their middle terms could not be polarized, or there was a lack of distribution, with the middle term not being connected to “All” or “No.” It’s also true that soriteses that begin with these invalid syllogistic forms will also be invalid. The question arises as to whether all of the 19 valid syllogistic forms are valid for their respective sorites, and the answer is no. The following table shows which soriteses in the form of the 19 valid syllogisms are in fact valid. The \( v \) stands for valid, while an empty space means the sorites is invalid:

<table>
<thead>
<tr>
<th>Name</th>
<th>Arist.</th>
<th>Gocl.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barbara</td>
<td>v</td>
<td>v</td>
</tr>
<tr>
<td>Celarent</td>
<td></td>
<td>v</td>
</tr>
<tr>
<td>Darii</td>
<td>v</td>
<td></td>
</tr>
<tr>
<td>Ferio</td>
<td></td>
<td>v</td>
</tr>
<tr>
<td>Cesare</td>
<td>v</td>
<td>v</td>
</tr>
<tr>
<td>Camestres</td>
<td>v</td>
<td>v</td>
</tr>
<tr>
<td>Festino</td>
<td>v</td>
<td></td>
</tr>
<tr>
<td>Baroko</td>
<td>v</td>
<td></td>
</tr>
<tr>
<td>Darapti</td>
<td>v</td>
<td>v</td>
</tr>
<tr>
<td>Disamis</td>
<td>v</td>
<td>v</td>
</tr>
<tr>
<td>Felapton</td>
<td>v</td>
<td></td>
</tr>
<tr>
<td>Bokardo</td>
<td>v</td>
<td></td>
</tr>
<tr>
<td>Ferison</td>
<td>v</td>
<td></td>
</tr>
<tr>
<td>Bramantip</td>
<td>v</td>
<td></td>
</tr>
<tr>
<td>Camenes</td>
<td>v</td>
<td>v</td>
</tr>
<tr>
<td>Dimaris</td>
<td>v</td>
<td></td>
</tr>
<tr>
<td>Fesapo</td>
<td>v</td>
<td>v</td>
</tr>
<tr>
<td>Fresison</td>
<td>v</td>
<td>v</td>
</tr>
</tbody>
</table>
5. The Sorites in Four Figures

In the following, I’ve taken each form and solved them in terms of whether they are valid or invalid. A proposition in parentheses represents the “hidden” conclusion that serves as the premiss of the next syllogism. Terms in bold print show where lack of distribution occurs.

6. Goclenian Sorites

The Goclenian Sorites in four figures:

<table>
<thead>
<tr>
<th>Table 9: First Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Barbara</strong></td>
</tr>
<tr>
<td>(1) ZaP</td>
</tr>
<tr>
<td>(2) YaZ</td>
</tr>
<tr>
<td>(3) (YaP)</td>
</tr>
<tr>
<td>(4) XaY</td>
</tr>
<tr>
<td>(5) (XaP)</td>
</tr>
<tr>
<td>(6) SaX</td>
</tr>
<tr>
<td>∴ Sap</td>
</tr>
<tr>
<td><strong>valid</strong></td>
</tr>
</tbody>
</table>

**Barbara:**
Ze~P + Ye~Z = YaP  
Ye~P + Xe~Y = XaP  
Xe~P + Se~X = Se~P or SaP

**Darii:**
Ze~P + Yo~Z = Yo~P or YiP  
YiP + XaY = ? “Y” not connected to “All” or “No.”

**Celarent:**
ZeP + Ye~Z = YeP  
YeP + Xe~Y = XeP  
XeP + Se~X = SeP

**Ferio:**
ZeP + Yo~Z = YiP  
YiP + XaY = ? “Y” not connected to “All” or “No.”

<table>
<thead>
<tr>
<th>Table 10: Second Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cesare</strong></td>
</tr>
<tr>
<td>(1) PaZ</td>
</tr>
<tr>
<td>(2) YaZ</td>
</tr>
</tbody>
</table>
Cesare:
ZeP + Ye~Z = YeP or PeY
YeP + Xe~Y = XeP or PeX
XeP + Se~X = SeP

Camestres:
~ZeP + YeZ = YeP
YeP + Xe~Y = XeP
XeP + Se~X = SeP

Festino:
ZeP + Yo~Z = YoP
YoP + XaY =? “Y” not connected to “All” or “No.”

Baroko:
~ZeP + YoZ = YoP
YoP + Xe~Y =? “Y” not connected to “All” or “No.”

Note, with Festino and Baroko, we could very well, if we chose, obvert and convert the conclusion of the first set of premises so that it lines up with the form of the second figure. I’ve left it in first figure form, however, so that the lack of distribution can be more easily seen. Nevertheless, if you want to place the conclusion in second figure form, you would need to obvert YoP to Yi~P, then convert Yi~P to ~PiY, then obvert ~PiY to ~Po~Y, and then the conclusion will be in second figure form. Take note that the rule about not being able to convert out of a particular negative only applies to premises, not to conclusions; and you can obvert and convert a conclusion to your heart’s desire. It’s true that the conclusion in this case, YoP, serves as the premiss for a further argument, but since that further argument must be in the second figure, the conclusion of the first premises must be adapted to the needs of the second set of premises. Hence, for sorites, we shall hold that if a “hidden” conclusion is particular negative, this particular negative takes on the “power” of any normal syllogistic conclusion, and we can represent it in terms of whatever figure the sorites is in.

<table>
<thead>
<tr>
<th>(3)</th>
<th>(PeY)</th>
<th>(PeY)</th>
<th>YoP</th>
<th>YoP</th>
</tr>
</thead>
<tbody>
<tr>
<td>(4)</td>
<td>XaY</td>
<td>XaY</td>
<td>XaY</td>
<td>XaY</td>
</tr>
<tr>
<td>(5)</td>
<td>(PeX)</td>
<td>(PeX)</td>
<td>(...)</td>
<td>(...)</td>
</tr>
<tr>
<td>(6)</td>
<td>SaX</td>
<td>SaX</td>
<td></td>
<td></td>
</tr>
<tr>
<td>:</td>
<td>SeP</td>
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<td>valid</td>
<td>invalid</td>
</tr>
</tbody>
</table>

Table 11: Third Figure:

<table>
<thead>
<tr>
<th>Darapti</th>
<th>Disamis</th>
<th>Datisi</th>
<th>Felapton</th>
<th>Bokardo</th>
<th>Ferison</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>ZaP</td>
<td>ZiP</td>
<td>ZeP</td>
<td>ZoP</td>
<td>ZeP</td>
</tr>
<tr>
<td>(2)</td>
<td>ZaY</td>
<td>ZaY</td>
<td>ZiY</td>
<td>ZaY</td>
<td>ZiY</td>
</tr>
<tr>
<td>(3)</td>
<td>(YiP)</td>
<td>(YiP)</td>
<td>(YiP)</td>
<td>(YeP)</td>
<td>(YoP)</td>
</tr>
<tr>
<td>(4)</td>
<td>YaX</td>
<td>YaX</td>
<td>YaX</td>
<td>YaX</td>
<td>YaX</td>
</tr>
<tr>
<td>(5)</td>
<td>(XiP)</td>
<td>(XiP)</td>
<td>(XiP)</td>
<td>(XoP)</td>
<td>(XoP)</td>
</tr>
<tr>
<td>(6)</td>
<td>XaS</td>
<td>XaS</td>
<td>XaS</td>
<td>XaS</td>
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</tbody>
</table>

13
Table 12: Fourth Figure:

<table>
<thead>
<tr>
<th></th>
<th>Bramantip</th>
<th>Camenes</th>
<th>Dimaris</th>
<th>Fesapo</th>
<th>Fresison</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>PaZ</td>
<td>PaZ</td>
<td>PiZ</td>
<td>PeZ</td>
<td>PeZ</td>
</tr>
<tr>
<td>(2)</td>
<td>ZaY</td>
<td>ZeY</td>
<td>ZaY</td>
<td>ZaY</td>
<td>ZiY</td>
</tr>
<tr>
<td>(3)</td>
<td>(PiY)</td>
<td>(PeY)</td>
<td>(PiY)</td>
<td>(YoP)</td>
<td>(YoP)</td>
</tr>
<tr>
<td>(4)</td>
<td>YaX</td>
<td>YaX</td>
<td>YaX</td>
<td>YaX</td>
<td>YaX</td>
</tr>
<tr>
<td>(5)</td>
<td>(XiP)</td>
<td>(XoP)</td>
<td>(XiP)</td>
<td>(XoP)</td>
<td>(XoP)</td>
</tr>
<tr>
<td>(6)</td>
<td>XaS</td>
<td>XaS</td>
<td>XaS</td>
<td>XaS</td>
<td>XaS</td>
</tr>
<tr>
<td></td>
<td>SiP</td>
<td>SoP</td>
<td>SiP</td>
<td>SoP</td>
<td>SoP</td>
</tr>
<tr>
<td>:.</td>
<td>valid</td>
<td>valid</td>
<td>valid</td>
<td>valid</td>
<td>valid</td>
</tr>
</tbody>
</table>

Bramantip:
Zo~P + Yo~Z = YiP or PiY
Yo~P + Xo~Y = XiP
Xo~P + So~X = SiP

Camenes:
~ZeP + Ye~Z = YeP or PeY
YeP + Xo~Y = XoP
XoP + So~X = SoP

Dimaris:
Zo~P + Yo~Z = YiP or PiY
Yo~P + Xo~Y = XiP
Xo~P + So~X = SiP

Fesapo:
ZeP + Yo~Z = YoP
YoP + Xo~Y = XoP
XoP + So~X = SoP

Fresison:
ZeP + Yo~Z = YoP
YoP + Xo~Y = XoP
XoP + So~X = SoP
7. Aristotelian Sorites

The Aristotelian sorites in four figures:

The Aristotelian sorites is a bit more complicated due to the switch of the major and minor premises from top to bottom. In writing an Aristotelian sorites, we need to switch the mnemonic letters, so that a sorites starting with…

SaX
XeY

...should be read backwards, so to speak. In other words the above sorites would have the letter e coming first, and the letter a coming second in the mnemonic. The above premises could in fact be the start of an Aristotelian sorites in Celarent of the first figure, not Camestres in the second figure. If we were to convert the above premises to normal form, with the major on top, then we would have…

XeY
SaX

... and it’s easy to see in the second set of premises why the sorites would actually be Celarent in the first figure (assuming, of course, that you are familiar with the figures of the standard 19 valid syllogisms). The following represent solutions to the Aristotelian sorites in the 19 forms of the syllogism:

<table>
<thead>
<tr>
<th>Table 13: First Figure:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Barbara</strong></td>
</tr>
<tr>
<td>(1)</td>
</tr>
<tr>
<td>(2)</td>
</tr>
<tr>
<td>(3)</td>
</tr>
<tr>
<td>(4)</td>
</tr>
<tr>
<td>(5)</td>
</tr>
<tr>
<td>(6)</td>
</tr>
<tr>
<td>:.</td>
</tr>
<tr>
<td><strong>valid</strong></td>
</tr>
</tbody>
</table>

*Barbara:*
Se¬X + Xe¬Y = SaY
Se¬Y + Ye¬Z = SaZ
Se¬Z + Ze¬P = SaP

*Celarent:*
Se¬X + XeY = SeY
SeY + Ye¬Z = ? Unpolarized

*Darii:*
So¬X + Xe¬Y = SiY
So¬Y + Y¬Z = SiZ
So¬Z + Ze¬P = SiP

*Ferio:*
So¬X + XeY = SoY
SoY + Ye¬Z = ? Unpolarized

15
### Table 14: Second Figure

<table>
<thead>
<tr>
<th></th>
<th>Cesare</th>
<th>Camestres</th>
<th>Festino</th>
<th>Baroko</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>SaX</td>
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<td>SiX</td>
<td>SoX</td>
</tr>
<tr>
<td>(2)</td>
<td>YeX</td>
<td>YaX</td>
<td>YeX</td>
<td>YaX</td>
</tr>
<tr>
<td>(3)</td>
<td>(SeY)</td>
<td>(SeY)</td>
<td>(SoY)</td>
<td>(SoY)</td>
</tr>
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<td>(4)</td>
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<td>ZaY</td>
<td>ZaY</td>
<td>ZaY</td>
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<tr>
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<tr>
<td>(6)</td>
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<tr>
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<td>SoP</td>
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<tr>
<td>.</td>
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<td></td>
<td>valid</td>
<td>valid</td>
</tr>
</tbody>
</table>

**Cesare:**

\[
S_o \neg X + X e \neg Y = S_e Y
\]

**Camestres:**

\[
S_e Y + \neg Y e \neg Z = S_e Z
\]

**Festino:**

\[
S_e Z + \neg Z e \neg P = S_e P
\]

**Baroko:**

\[
S_o \neg X + X e \neg Y = S_e Y
\]

### Table 15: Third Figure

<table>
<thead>
<tr>
<th></th>
<th>Darapti</th>
<th>Disamis</th>
<th>Datisi</th>
<th>Felapton</th>
<th>Bokardo</th>
<th>Ferison</th>
</tr>
</thead>
<tbody>
<tr>
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<td>XiS</td>
<td>XaS</td>
<td>XaS</td>
<td>XiS</td>
</tr>
<tr>
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<td>XoY</td>
<td>XeY</td>
</tr>
<tr>
<td>(3)</td>
<td>(YiS)</td>
<td>(YiS)</td>
<td>(YiS)</td>
<td>(SoY)</td>
<td>(SoY)</td>
<td>(SoY)</td>
</tr>
<tr>
<td>(4)</td>
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<tr>
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<tr>
<td>(6)</td>
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<tr>
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<td>invalid</td>
</tr>
</tbody>
</table>

**Darapti:**

\[
S_o \neg X + X e \neg Y = S_i Y
\]

**Disamis:**

\[
S_o \neg X + X e \neg Y = S_i Y
\]

\[
S_o \neg Y + \neg Y e \neg Z = S_i Z
\]

**Datisi:**

\[
S_o Z + Z e \neg P = S_i P
\]

**Felapton:**

\[
S_o \neg X + X e \neg Y = S_i Y
\]

\[
S_o \neg Y + \neg Y e \neg Z = S_i Z
\]

**Bokardo:**

\[
S_o \neg X + X e \neg Y = S_i Y
\]

**Ferison:**

\[
S_o \neg X + X e \neg Y = S_i Y
\]
Felapton:
So¬X + XeY = SoY
SoY + Ye¬Z = ? Unpolarized

Ferison:
So¬X + XeY = SoY
SoY + Ye¬Z = ? Unpolarized

Bokardo:

<table>
<thead>
<tr>
<th>Table 16: Fourth Figure:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bramantip</td>
</tr>
<tr>
<td>(1)</td>
</tr>
<tr>
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<td>(4)</td>
</tr>
<tr>
<td>(5)</td>
</tr>
<tr>
<td>(6)</td>
</tr>
<tr>
<td>∴</td>
</tr>
</tbody>
</table>

Bramantip:
So¬X + Xo¬Y = SiY
So¬Y + Yo¬Z = ? Not connected to “All” or “No.”

Camenes:
SeX + ¬XeY = SeY
SeY + ¬YeZ = SeZ
SeZ + ¬ZeP = SeP

Dimaris:
So¬X + XeY = SoY
SoY + ¬YeZ = SoZ
SoZ + ¬ZeP = SoP

Fesapo:
So¬X + XoY = SoY
SoY + ¬YeZ = SoZ
SoZ + ¬ZeP = SoP

Fresison:
So¬X + XeY = SoY
SoY + ¬YeZ = SoZ
SoZ + ¬ZeP = SoP